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This study toward quantum gravity (QG) introduces an $SU(N)$ gauge theory with the Θ vacuum term as a trial theory. Newton gravitation constant G_N is realized as the effective coupling constant for a massive graviton, $G_N/\sqrt{2} = g_f g_g^2/8M_G^2 \approx 10^{-38}$ GeV^{-2} with the gauge boson mass $M_G = M_{Pl} \approx 10^{19}$ GeV, the gravitational coupling constant g_g, and the gravitational factor g_f. This scheme postulates the effective cosmological constant as the effective vacuum energy represented by massive gauge bosons, $\Lambda_e = 8\pi G_N M_G^4$, and provides a plausible explanation for the small cosmological constant at the present epoch $\Lambda_0 \approx 10^{-84}$ GeV^2 and the large value at the Planck epoch $\Lambda_{Pl} \approx 10^{38}$ GeV^2; the condensation of the singlet gauge field $\langle\phi\rangle$ triggers the current anomaly and subtracts the gauge boson mass, $M_G^2 = M_{Pl}^2 - g_f g_g^2 \langle\phi\rangle^2 = g_f g_g^2 (A_0^2 - \langle\phi\rangle^2)$, as the vacuum energy. Relations among QG, general relativity, and Newtonian mechanics are discussed.

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I. INTRODUCTION

Einstein's general theory for relativity [1] is a presently accepted, classical theory for gravitation and is tested in the weak field approximation. However, general relativity or standard big bang theory based on general relativity has outstanding problems: the singularity, cosmological constant or vacuum energy, flat universe, baryon asymmetry, horizon problem, the large scale homogeneity and isotropy of the universe, dark matter, galaxy formation, discrepancy between astrophysical age and Hubble age, etc. These problems might be resolved and the eventual unification of fundamental forces might be possible if quantum theory for gravitation is found. General relativity is to be replaced by quantum theory for gravitation in order to unify gravity with the other fundamental forces. Nevertheless, there is so far no satisfactory quantum alternative for general relativity and accordingly no quantum cosmology for standard big bang theory. The longstanding problems in physics are thus abstracted as the quantization of gravity, the cosmological constant problem, and the unification of gravitation with the other fundamental forces. These problems are the motivations for quantum gravity (QG) interested in this paper. According to recent experiments, BUMERANG-98 and MAXIMA-1 [2], the universe is flat and there exists repulsive force represented by non-zero vacuum energy, which plays dominant role in the universe expansion. In this paper and subsequent paper [3], the quantum and classical features of QG as a gauge theory associated with a group G are suggested to resolve the theoretical problems in general relativity and are justified by the experimental facts even though the group G is exactly not known at present: the group chain is given by $G \supset SU(2)_L \times U(1)_Y \times SU(3)_C$ where G, $SU(2)_L \times U(1)_Y$, and $SU(3)_C$ groups are for gravitation, weak [4], and strong [5] interactions respectively. Specifically, an $SU(N)$ gauge theory as a trial theory toward QG is introduced to resolve topics relevant for Newton gravitation constant G_N and the cosmological constant and to achieve the possible generation of all the fundamental interactions in terms of dynamical spontaneous symmetry breaking (DSSB). This paper also tries to illustrate classical tests verified by Einstein's general relativity: quantum tests predicted by gauge theories, from the Planck scale 10^{-33} cm to the universe scale 10^{28} cm, are discussed in subsequent paper [3]. The present work is mainly restricted to the low, real dimensions of space-time without considering supersymmetry. This work is based on phenomenology below the Planck scale: Newtonian mechanics, Einstein's general relativity, and quantum gauge theories.

Einstein successfully developed the classical field equation known as general relativity theory for gravity [1], which arises naturally from a systematic and explicit account of the geometrical structure of space and time as dynamic constituents of nature, affected by matter or energy. The standard cosmology based on the Einstein's general theory of relativity is the hot big bang model. This model is supplemented by the success of the grand unified theory (GUT) [6] based on quantum gauge theory, that is able to show how the universe evolved since the initial big bang. A logical consequence of the GUT phase transition is inflation which promises to resolve some of its outstanding problems, flatness and horizon problems. Quantum gauge theory which is the marriage of quantum theory and gauge symmetry has been indispensable in providing a precise description of the microcosm. Three fundamental forces except gravity are described by quantum gauge theory; the $U(1)_e$ gauge theory for electromagnetic force is one of the most successful theory in physics at the moment and the $SU(2)_L \times U(1)_Y$ gauge theory for weak force [4] and the $SU(3)_C$ gauge theory for strong force [5] are non-Abelian extensions of the $U(1)_e$ gauge theory. These two fundamental theories, general relativity and quantum gauge theory, can together cover the energy regime from the elementary particle to the universe in describing the behavior of particles. However, the outstanding problem of unifying gravity and the other fundamental forces in nature is that general relativity theory is difficult to formulate as a renormalizable gauge theory. All the known covariant or canonical quantization methods are not yet successful in quantizing the gravitational field, even though they work well for the other physical fields. There is no distinct connection between Einstein's general relativity on which the standard cosmology is based and gauge theory on which GUT is based. The incompatibility of the two modern theories, general relativity and gauge theory, is thus the biggest obstacle to the unification of the two theories into one theoretical framework; as known widely, the unification of the two theories has been one of the greatest challenge in physics. There are usually two directions toward quantum gravitation theory or the unification of fundamental forces: superstring theory and Kaluza-Klein theory in the higher dimensions and the Planck scale. Superstring theory is considered to be one of the most promising candidate in the unification of forces but there has been no known compactification method to break down to the real, low energy world and no clear answer to how superstring theory solves the cosmological constant problem.

Another longstanding problem in cosmology is so-called the cosmological constant problem since it has been introduced by Einstein [7]. The cosmological constant is regarded as one of the most fundamental physical entities and various attempts, which include supersymmetry, anthropic consideration, adjustment mechanics, changing gravity, and quantum gravity, have been made but none of them is completely able to resolve the problem [8]. The effective cosmological constant is very small or nearly zero according to the astronomical observation while the several theories demand that the cosmological constant should be relatively large [8]. The discrepancy in the cosmological constant or vacuum energy density 10^{122} GeV^4 between the experimental observation and the theoretical expectation are, on the one hand, more enormous than any other physical quantities. On the other hand, according to recent experiments, BUMERANG-98 and MAXIMA-1 [2], the universe is flat and there exists repulsive force represented by non-zero vacuum energy, which plays dominant role in the universe expansion. This phenomena is contradicted with general relativity which is accepted, classical theory for gravitation. In this context, it is quite natural to develop quantum gauge theory to overcome these theoretical problems and to explain experimental observations.

An $SU(N)$ gauge theory with the Θ vacum toward QG is in this paper introduced as a trial theory to resolve topics relevant for Newton gravitation constant G_N and the cosmological constant and illustrates the possible classical and quantum tests. Newtonian gravitation constant G_N, which is regarded as one of fundamental constants, is postulated as the effective coupling constant like Fermi constant G_F in weak interaction [4]. The dimensionality of G_N, which is the obstacle of renormalization, is overcome by introducing the gravitational coupling constant α_g and the massive gauge boson (graviton) with the Planck mass $M_G = M_{Pl}$. The cosmological constant is also defined in terms of the gauge boson mass by the effective cosmological constant $\Lambda_e = 8\pi G_N M_G^4$, which varies from the huge value $\Lambda_e = \Lambda_{Pl} = 10^{76}$ GeV^2 at the Planck scale to the measured, small value $\Lambda_e = \Lambda_0 = 10^{-84}$ GeV^2 at the present universe. In order to test QG, classical tests predicted by general relativity and verified by observations are here considered and quantum tests predicted by a quantum gauge theory are discussed in the subsequent paper [3]. Newtonian mechanics is postulated as quadrupole interactions of QG. Among classical tests, the deflection of light by the Sun and the precession of the perihelia of the orbits of the inner planets are the typical tests carried out both in empty space and in gravitational fields that are approximately static and spherically symmetric [9]. The energy per mass, which exhibits the classical tests, obtained from Einstein's general relativity in the weak field limit are therefore examined from the viewpoints of QG as a gauge theory.

This paper is organized as follows. In Section II, an $SU(N)$ gauge theory toward QG is suggested without considering supersymetry and then DSSB is introduced. Section III describes the origin of Newton gravitation constant and the dual Meissner effect presented as a plausible reason for no detection of gravitational waves in this scheme. Section IV deals with the resolution of the cosmological constant problem as the vacuum energy density. In Section V, classical tests for gravity are discussed from the potential QG point of view, in which Newtonian mechanics is postulated as a quadrupole interaction of QG. Section VI addresses comparison between QG and general relativity. Section VII is devoted to conclusions.

II. TOWARD QUANTUM GRAVITY

An $SU(N)$ gauge invariant Lagrangian density with the Θ vacuum term is, without taking into account supersymmetry, introduced toward QG as a trial theory even though the exact group G for gravity is not unveiled. DSSB triggered by the Θ term is adopted to generate gauge boson mass and fermion mass. Natural units with $\hbar = c = k_B = 1$ are preferred for convenience throughout this paper unless otherwise specified.

The gauge invariant Lagrangian density is, in four vector notation, given by

$$\mathcal{L} = -\frac{1}{2} Tr G_{\mu\nu} G^{\mu\nu} + \sum_{i=1} \bar{\psi}_i i\gamma^\mu D_\mu \psi_i \qquad (1)$$

where the subscript i stands for the classes of pointlike spinors, ψ for the spinor, and $D_\mu = \partial_\mu - ig_g A_\mu$ for the covariant derivative with the gravitational coupling constant g_g. Particles carry the local charges and the gauge fields are denoted by $A_\mu = \sum_{a=0} A_\mu^a \lambda^a/2$ with matrices λ^a, $a = 0, .., (N^2 - 1)$. The field strength tensor is given by $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_g[A_\mu, A_\nu]$. A current anomaly

[10] is taken into account to show DSSB in analogy with the axial current anomaly, which is linked to the Θ vacuum in QCD as a gauge theory [11]. The bare Θ term is added as a single, additional nonperturbative term to the Lagrangian density (1)

$$\mathcal{L}_{QG} = \mathcal{L}_P + \Theta \frac{g_g^2}{16\pi^2} TrG^{\mu\nu}\tilde{G}_{\mu\nu}, \tag{2}$$

where $\mathcal{L}_P$ is the perturbative Lagrangian density (1), $G^{\mu\nu}$ is the field strength tensor, and $\tilde{G}_{\mu\nu}$ is the dual of the field strength tensor. Since the $G\tilde{G}$ term is a total derivative, it does not affect the perturbative aspects of the theory.

DSSB consists of two simultaneous mechanisms; the first mechanism is the explicit symmetry breaking of gauge symmetry, which is represented by the gravitational factor g_f and the gravitational coupling constant g_g, and the second mechanism is the spontaneous symmetry breaking of gauge fields, which is represented by the condensation of gravitational singlet gauge fields. Gauge fields are generally decomposed by charge nonsinglet-singlet on the one hand and by even-odd discrete symmetries on the other hand: they have dual properties in charge and discrete symmetries. Four singlet gauge boson interactions in (2), apart from nonsinglet gauge bosons, are parameterized by the $SU(N)$ symmetric scalar potential:

$$V_e(\phi) = V_0 + \mu^2\phi^2 + \lambda\phi^4 \tag{3}$$

which is the typical potential with $\mu^2 < 0$ and $\lambda > 0$ for spontaneous symmetry breaking. The first term of the right hand side corresponds to the vacuum energy density representing the zero-point energy by even parity singlets. The odd-parity vacuum field ϕ is shifted by an invariant quantity $\langle\phi\rangle$, which satisfies

$$\langle\phi\rangle^2 = \phi_0^2 + \phi_1^2 + \cdots + \phi_N^2 \tag{4}$$

with the condensation of odd-parity singlet gauge bosons: $\langle\phi\rangle = (\frac{-\mu^2}{2\lambda})^{1/2}$. DSSB is relevant for the surface term $\Theta\frac{g_g^2}{16\pi^2}TrG^{\mu\nu}\tilde{G}_{\mu\nu}$, which explicitly breaks down the $SU(N)$ gauge symmetry for quantum gravity through the condensation of odd-parity singlet gauge bosons. The Θ can be assigned by an dynamic parameter by

$$\Theta = 10^{-61}\ \rho_G/\rho_m \tag{5}$$

with the matter energy density ρ_m and the vacuum energy density $\rho_G = M_G^4$.

III. NEWTON GRAVITATION CONSTANT

The longstanding problems of both quantum gauge theory for gravity and the cosmological constant problem might be resolved through DSSB. Although the exact group is not known at a moment, the features of quantum gravity as a gauge theory may be suggested and the origin of Newton gravitation constant G_N may be discussed. The group chain $G \supset SU(2)_L \times U(1)_Y \times SU(3)_C$ and the effective coupling constant chain $G_N \supset G_F \times G_R$ are introduced [4,13,5]. In the following, Newton gravitation constant and graviton mass, coupling constants for fundamental forces, and the dual Meissner effect are addressed.

A. Newton Gravitation Constant and Graviton Mass

Gravitational interactions are generated by the emission and absorption of vector bosons with spin 1, gravitons, rather than tensor bosons with spin 2. Gravitons are the analogs of photons for electromagnetic force and gluons for color force. Newton gravitation constant G_N is cast in a form that can be directly compared to Fermi coupling constant G_F of electroweak interactions. G_N replaces graviton propagation $\sqrt{2}g_f g_g^2/8(k^2 - M_G^2)$ with the graviton mass M_G and, in contrast to the dimensionless coupling constant g_g and the gravitational factor g_f, has the dimension of inverse energy square. Contrary to the photon, the graviton must be massive, otherwise it would have been directly produced in the gravitational decays. It, in fact, turns out that the graviton has the Planck mass $M_G \simeq M_{Pl} = 1.22 \times 10^{19}$ GeV.

The gravitational interaction amplitude below the Planck energy is of the form

$$\mathcal{M} = -\frac{g_f g_g^2}{4} J^\mu \frac{1}{k^2 - M_G^2} J_\mu^\dagger = \sqrt{2}G_N J^\mu J_\mu^\dagger \tag{6}$$

where $\mathcal{M}$ is the product of two universal current densities and g_f is the gravitational factor, which is defined by $g_f = \frac{1}{4}(g_3^\dagger\lambda^a g_1)(g_2^\dagger\lambda_a g_4)$ with the gravitational charge fields, g_i with $i = 1 \sim 4$, in analogy with the color factor c_f in QCD. The current density J^μ will be discussed with relation to the cosmological constant and the baryon asymmetry in the following sections. If $k^2 << M_G^2$, the effective gravitational coupling becomes

$$\frac{G_N}{\sqrt{2}} = -\frac{g_f g_g^2}{8(k^2 - M_G^2)} \simeq \frac{g_f g_g^2}{8M_G^2} \simeq 10^{-38}\ \text{GeV}^{-2} \tag{7}$$

and the gravitational currents interact essentially at a point. That is, in the low momentum transfer, the propagation between the currents disappears.

Gravitational gauge field has the Planck mass at the phase transition:

$$M_G \approx M_{Pl} \approx 10^{19}\ \text{GeV} \tag{8}$$

which is reduced to a smaller value due to the condensation of the singlet graviton. Note that the conventional relation $G_N = 1/M_{Pl}^2$ is adjusted to $G_N \simeq \sqrt{2}g_f g_g^2/8M_{Pl}^2$ in (7). The gauge boson mass below the Planck energy can be cast by

$$M_G^2 = M_{Pl}^2 - g_f g_g^2\langle\phi\rangle^2 = g_f g_g^2[A_0^2 - \langle\phi\rangle^2] \tag{9}$$

with the even parity singlet gauge boson A_0, the odd-parity singlet gauge boson condensation $\langle\phi\rangle$, the gravitational charge factor g_f, and the coupling constant g_g. The gravitational factor g_f used in (9) is the symmetric factor for a gauge boson with even parity and the asymmetric factor for a gauge boson with odd parity. This process makes the breaking of discrete symmetries P, C, T, and CP. The effective vacuum energy density $V_e(\bar{\phi})$ in (3) is connected with the gauge boson mass M_G by $V_e = M_G^4$: $V_0 = M_{Pl}^4$, $\mu^2 = -2g_f g_g^2 M_{Pl}^2$, $\lambda = g_f^2 g_g^4$. The gauge boson condensation subtracting the gauge boson mass is related to the subtraction of the zero-point energy in the system. Newton gravitation constant at low momentum transfer thus can be related to the graviton mass by expression (7). The gauge boson mass must be identical to the inverse of the screening length, that is, $M_G = 1/l_{Pl} \simeq G_N^{1/2}$. A graviton thus has the enormous mass $M_G \approx 10^{19}$ GeV and its propagation is prevented from observation for the low energy graviton; this is known as the dual Meissner effect. This is the reason why gravitational force is phenomenologically so weak compared to the other forces and gravitational wave is not detected.

The above description is illustrated more rigorously by considering the Yukawa potential due to the massive graviton; the Coulomb potential for the energetic gauge boson is screened by the mass of the gauge boson and consequently the effective gravitation coupling is very weak at the macroscopic scale. The Yukawa potential associated with the massive gauge boson is, in the static limit $E \to 0$,

$$V(r) = \sqrt{\frac{g_f g_g^2}{4\pi}} \frac{e^{-M_G(r-l_{Pl})}}{r} \tag{10}$$

which represents the short range interaction for the low energy graviton.

QG as a gauge theory with a certain group G has gauge bosons, gravitons, whose interactions depend on their energies and masses. The graviton with the energy higher than the Planck mass propagates like massless particle, the graviton with the energy equal to the Planck mass shows the Newtonian interaction, and the graviton with the energy lower than the Planck mass propagates only inside the Planck scale. G_N thus makes two types of potentials in gravitational coupling; the Yukawa potential for the graviton with the energy lower than the Planck mass ($E < M_{Pl}$) and the constant potential for the graviton with the energy equal to the Planck mass ($E \to M_{Pl}$). This implies that there are generally two kinds of potentials for massive gauge bosons due to the boundary condition; one is the constant gravitational potential for $E \to M_G$ and the other is the Yukawa type gravitational potential for $E < M_G$. The graviton with the energy lower than the Planck mass has the Yukawa potential $\propto e^{-M_G^e(r-l_{Pl})}$ with the Planck length $l_{Pl} = 1/M_{Pl}$ and the effective mass $M_G^e = ik_G = (M_G^2 - E^2)^{1/2}$, the graviton with the energy higher than the Planck mass has outgoing propagation $\propto e^{ikr}$ with $k = (E^2 - M_G^2)^{1/2}$, the graviton with energy equal to the gauge boson mass ($E \to M_G$) has the constant potential. This suggests that the effective interaction of the graviton with the energy below the Planck energy is proportional to $G_N J_\mu J^{\mu\dagger}$ in the low momentum transfer. Below the Planck energy, a phase transition takes place from a large group G to the $SU(2)_L \times U(1)_Y \times SU(3)_C$ group, where the $SU(2)_L \times U(1)_Y$ denotes the weak isospin symmetry and the $SU(3)_C$ denotes the color symmetry, via a unification group H, that is, $G \supset H \supset SU(2)_L \times U(1)_Y \times SU(3)_C$. Massive gravitons may be connected to intermediate vector bosons and photons at the electroweak scale and to gluons and photons at the strong scale.

B. Coupling Constants for Fundamental Forces

Out of four fundamental forces in nature, the most familiar forces are gravity and electromagnetism which are experienced in the macroscopic scale. Strong interactions are limited in range to about 10^{-13} cm and are insignificant even at the scale of the atom 10^{-8} cm, but play an important role in the binding the nucleus. Weak interactions with an even shorter scale ($\leq 10^{-15}$ cm) are so weak that they do not bind anything, but they do play an important role in weak decay processes. Such local interactions of strong and weak forces are connected with their massive gauge bosons; for example, weak force is restricted by the intermediate vector boson mass M_W with the effective coupling constant $G_F = \sqrt{2} i_f g_i^2/8M_G^2$. Newton gravitation constant due to the massive graviton is the origin of all the effective coupling constants [4,13]; the effective coupling constant chain is $G_N \supset G_F \times G_R$ with Fermi weak constant $G_F \approx 10^{-5}$ GeV^{-2} and the effective strong coupling constant $G_R = \sqrt{2} c_f g_s^2/8M_G^2 \approx 10$ GeV^{-2}. Strengths of four forces are roughly in orders of magnitude $10, 10^{-2}, 10^{-5}$, and 10^{-40} for strong, electromagnetic, weak, and gravitational forces respectively. The difference in strength is more than a factor of $G_R/G_N \approx 10^{39}$ between strong and gravitational interactions. From the ratio of $G_F/G_N \approx 10^{33}$, electroweak force is 10^{33} times stronger than gravitational force. In gravitational interactions, it is predicted that the typical cross section at a temperature $T = 1$ GeV is $\sigma \simeq G_N^2 T^2 \approx 10^{-70}$ m^2 and the typical lifetime for a particle with the mass 1 GeV is $\tau = 1/\Gamma \simeq 1/G_N^2 m^5 \approx 10^{50}$ years. Similarly, in the proton decay $p \to \pi^0 + e^+$ at the energy $E << M_{Pl}$, the proton would have much longer lifetime than 10^{30} years if the decay process is gravitational. Features of fundamental interactions are summarized in Table I.

A phase transition at the Planck scale creates massive gravitons and massless gauge bosons like the Higgs mechanism of electroweak interactions [12]; there is mixing between gravitation currents so as to produce massive gravitons and massless gauge bosons as Nambu-Goldstone

(NG) bosons [14]. Massive gravitons produce the Yukawa gravitational potential with Newton gravitation constant outside the Planck scale for the graviton with the energy $E < M_{Pl}$. Massless gauge bosons (photons) produce the Coulomb potential in the static limit and have a weaker coupling constant than the electromagnetic coupling constant $\alpha_e = 1/137$ for the photon. There is therefore the possibility that massless gauge bosons (photons) with the energy $E_\gamma \approx 10^{19}$ GeV $\approx 10^{42}$ Hz responsible for the $U(1)$ gauge theory were left as relics of phase transition at the Planck era.

One possibility as a candidate of quantum gravity is an $SU(3)_G$ gauge theory under the assumption of no supersymmetry and higher dimensions. The phase transition of $SU(3)_G \to SU(2) \times U(1) \to U(1)_c$ takes place at the Planck energy just as $SU(3)_I \to SU(2)_L \times U(1)_Y \to U(1)_e$ for weak force [15,4] and $SU(3)_C \to SU(2)_N \times U(1)_Z \to U(1)_f$ for strong force [13]. In this case, the groups for all the fundamental forces hold the $SU(3)$ gauge symmetry as an essentially analogous dynamics although each fundamental force dominates at different energy; the gauge group hierarchy is $SU(3)_G \supset SU(3)_I \supset SU(3)_C$ and the effective coupling constant hierarchy is $G_N \supset G_F \supset G_R$. At higher energies, weakly interacting massive particles are relatively dominant but at lower energies, strongly interacting massive particles are relatively dominant. In terms of the renormalization group analysis, the considerable coupling constant for gravitational interactions $\alpha_g \simeq 0.019$ is extracted from the weak and strong coupling constant $\alpha_i = \alpha_s \simeq 0.12$ around the unification energy 10^2 GeV when three charges and six flavors are assumed. The massless gauge boson for the $U(1)_c$ gauge theory might have the coupling constant $\alpha_g/8 \simeq 1/410$ for attractive interactions, which is about one third of the electromagnetic coupling constant $\alpha_e = 1/137$ for the $U(1)_e$ gauge theory. Whatever the group G is, a test for this possibility will thus be the search of the massless gauge boson (photon) with an extremely high frequency 10^{42} Hz.

C. Dual Meissner Effect in Gravitational Waves

General relativity shows the existence of gravitational waves but they have not been detected so far. The plausible reason why gravitational waves have not been observed is more focused on in this part. DSSB is undergoing through the condensation of the singlet graviton in the evolution stage of the universe so as to produce the gravitational superconducting state in the vacuum. Below the Planck energy, the condensation of the singlet gauge field increases as the universe expands and particles make the Cooper pairs in the superconducting state. The essential point is that the gauge field acquires a vacuum expectation value and the resulting phase transition is still underway in the present universe.

In this case, the difficulty in the detection of gravitational waves is easily overcome by the dual Meissner effect in gravitational waves, which is analogous to the Meissner effect in the electric superconductivity [16]: the exclusion of the gravitational electric field in the superconducting state outside the Planck scale. The evolution of the universe makes DSSB during which the condensation of the singlet gauge field proceeds, the Cooper pair of particles is formed, and flux tube is made by the gravitational electric field. The gravitational electric field is therefore excluded in the gravitational superconducting state due to the dual Meissner effect; this is interpreted by the fact that the graviton with the energy lower than the Planck mass becomes massive. If the Cooper pairs due to the gravitational magnetic field in the vacuum are made, the exclusion of the gravitational electric field is obvious in the present universe. The extremely massive graviton therefore provides the reason why gravitational wave has not been detected and has the effective weak coupling for gravity G_N. This argument is dually analogous to the screening mechanism of magnetic fields in the presence of electric superconductor [17]. More discussion on the dual Meissner effect is addressed in the topics of fermion mass generation and classical tests of QG. On the other hand, the massless gauge boson created during DSSB at the Planck epoch has the enormous frequency 10^{42} Hz as described above.

IV. COSMOLOGICAL CONSTANT

The outstanding problem of the cosmological constant may be resolved from the view point of gauge theory for gravity, in which the cosmological constant is regarded as the vacuum energy represented by the gauge boson mass. The constraint of the flat universe, $\Omega - 1 = 10^{-61}$, is required by quantum gauge theory and is confirmed by experiments BUMERANG-98 and MAXIMA-1 [2]. The vacuum represented by the massive gauge bosons is quantized by the maximum wavevector mode $N_R = i/(\Omega - 1)^{1/2} \approx 10^{30}$ and the total gauge boson number $N_G = 4\pi N_R^3/3 \approx 10^{91}$; the gauge boson mass M_G is quantized by the maximum wavevector mode N_R. DSSB takes place through the condensation of singlet gauge bosons leading to the current anomaly and decreases the effective cosmological constant. The effective cosmological constant is here defined by $\Lambda_e = 8\pi G_N M_G^4$: $\Lambda_e \approx M_{Pl}^2 \approx 10^{38}$ GeV^2 at the Planck epoch and $\Lambda_e = \Lambda_0 = 3H_0^2 \approx 10^{-84}$ GeV^2 at the present epoch. The effective cosmological constant $\Lambda_e = \Lambda_0 = \Lambda(t = t_0)$ defined at the present epoch $t = t_0$ is a number with unit cm^{-2}, which is independent of particle mass and Newton gravitation constant G_N, and the cosmological interaction occurs even in the absence of any matter at all.

In the following, the cosmological constant problem, the resolution of the cosmological constant, and the modification of general relativity are described.

A. The Cosmological Constant Problem

The universal cosmological constant Λ can be inserted in Einstein's field equation for gravity without destroying the general covariance:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G_N \tilde{T}_{\mu\nu} = -8\pi G_N T_{\mu\nu} + g_{\mu\nu}\Lambda \quad (11)$$

where $R_{\mu\nu}$ denotes the Ricci tensor, $R = R^{\nu}_{\nu}$ the scalar curvature, G_N Newton gravitation universal constant, $T_{\mu\nu}$ the energy momentum tensor with its trace $T = T^{\mu}_{\mu}$, $\tilde{T}_{\mu\nu}$ the total energy-momentum tensor, and Λ the bare cosmological constant. From Einstein's field equation the effective cosmological constant takes the form

$$\Lambda_e = \Lambda + 8\pi G_N \langle\rho_m\rangle \quad (12)$$

where $-\langle\rho_m\rangle g_{\mu\nu} = \langle T_{\mu\nu}\rangle$ is the energy-momentum tensor in a vacuum according to Lorentz invariance. The total effective vacuum energy is expressed by $\langle\rho_m\rangle_e = \langle\rho_m\rangle + \Lambda/(8\pi G_N)$. Measurements of cosmological redshifts as a function of distance provide the upper bound on $\Lambda_e < H_0^2$ or $\langle\rho_m\rangle_e < 8.07 \times 10^{-47}$ GeV^4 with the Hubble constant H_0 [18]. If the vacuum energy density $\langle\rho_m\rangle$ is approximated by $\langle\rho_m\rangle \approx \Lambda^4_{cut}/(16\pi^2)$ with the wave number cutoff Λ_{cut}, $\langle\rho_m\rangle = 2 \times 10^{71}$ GeV^4 in the case of $\Lambda_{cut} = (8\pi G_N)^{-1/2}$. Since $\langle\rho_m\rangle_e$ is less than 10^{-47} GeV^4 two terms, $\langle\rho_m\rangle$ and $\Lambda/(8\pi G_N)$, must cancel to better than 118 decimal places. This is so-called the cosmological constant problem [8].

The cosmological constant is also related to the vacuum energy density $V(\bar{\phi})$ [19] described by a constant scalar field $\bar{\phi}$. When the scalar field appears, the change in the vacuum energy density $V(\bar{\phi})$ enters into Einstein's field equation

$$R_{\mu\nu} - 1/2 g_{\mu\nu}R = -8\pi G_N \tilde{T}_{\mu\nu} = -8\pi G_N (T_{\mu\nu} - g_{\mu\nu}V(\bar{\phi})) \quad (13)$$

where $g_{\mu\nu}V(\bar{\phi})$ is the energy-momentum tensor of the vacuum. Comparing the energy-momentum tensor $T_{\mu\nu}$ with $g_{\mu\nu}V(\bar{\phi})$, the pressure exerted by the vacuum and the energy density have opposite sign, $P_\Lambda = -\rho_\Lambda = -V(\bar{\phi})$. The vacuum energy density multiplied by $8\pi G_N$ is usually called the cosmological constant Λ, that is, $\Lambda = 8\pi G_N V(\bar{\phi})$. From equation (13) in the Minkowski coordinates, the cosmological constant thus acts like a fluid with the effective mass density $\rho_\Lambda = \Lambda/(8\pi G_N)$ and the pressure $P_\Lambda = -\rho_\Lambda$.

The consequence of a non-zero value for the cosmological constant has been extensively discussed [8,19] in the context of dynamics of the universe. The cosmological constant prevents static universe from collapsing against gravity since its positive sign plays as a universal repulsive force for space. In the inflation scenario [20], a scalar field with a non-zero vacuum energy density serves as a driving force for the exponential expansion. This can be interpreted as that the effective cosmological constant would be a large positive constant in the early universe, which might reduce to its present small positive value after series of phase transitions in the early universe.

The cosmological constant problem or equivalently the vacuum energy problem is resolved in the following subsection from the gauge theory point of view.

B. Resolution of the Cosmological Constant

QG as a gauge theory exhibits reasonable explanations to the known phenomenological, cosmological constant problem; the effective cosmological constant Λ_e is connected with the effective vacuum energy density $V_e(\bar{\phi}) = \langle\rho_m\rangle_e$ linked by the gauge boson mass M_G. For the gauge boson with the energy E higher than its mass M_G, the interaction can be overall repulsion before the phase transition and for the gauge boson with the energy lower than its mass, the interaction is overall attraction after the phase transition in the matter space. The effective cosmological constant Λ_e representing the effective vacuum energy density decreases its value through DSSB from a false vacuum to a true vacuum, induced by the condensation of singlet gravitons. A vacuum state is stable if vacuum expectation values for gauge bosons are at a true minimum of the effective potential. However, if vacuum expectation values are at a local minimum that is higher than the true minimum, then this vacuum will be metastable. A metastable, false vacuum state corresponding to a local minimum above the Planck energy will decay into the stable true vacuum corresponding to the true minimum by a tunneling process. The normal vacuum or false vacuum has several characteristics: isotropy, homogeneity, degeneracy, etc. as much as the number of directions. The effective cosmological constant Λ_e represented by the graviton mass M_G decreases through the DSSB mechanism, which is triggered by the condensation of the singlet gauge field.

The extremely small deviation of the flat universe,

$$\Omega - 1 = 10^{-61}, \quad (14)$$

is required by quantum gauge theory, is confirmed by experiments BUMERANG-98 and MAXIMA-1 [2], and is constrained by the space time quantization in the order of 10^{30} in one dimension. The vacuum represented by massive gauge bosons is thus quantized by the maximum wavevector mode $N_R = i/(\Omega-1)^{1/2} \approx 10^{30}$ and the total gauge boson number $N_G = 4\pi N_R^3/3 \approx 10^{91}$; the effective gauge boson mass $M_G^e = ik_G = (M_G^2 - E^2)^{1/2}$ is quantized by the maximum wavevector mode N_R. The effective cosmological constant is defined by $\Lambda_e = 8\pi G_N M_G^4$ where the gauge boson mass square is given by $M_G^2 = M_{Pl}^2 - g_f g_g^2 \langle\phi\rangle^2 = g_f g_g^2 [A_0^2 - \langle\phi\rangle^2]$ below the Planck energy. The effective cosmological constants are respectively calculated from the graviton mass $M_G \approx 10^{19}$ GeV at the Planck era and the nearly massless gauge boson mass $M_G \approx 10^{-12}$ GeV at the present era: the effective cosmological constants are $\Lambda_{Pl} > \Lambda_{EW} > \Lambda_{QCD} > \Lambda_0$, where $\Lambda_e = \Lambda_{Pl} = M_{Pl}^2 \approx 10^{38}$ GeV^2 at the Planck epoch, $\Lambda_e = \Lambda_{EW} \approx 10^{-30}$ GeV^2 at the weak epoch, $\Lambda_e = \Lambda_{QCD} \approx 10^{-42}$ GeV^2 at the strong epoch, and $\Lambda_e = \Lambda_0 = 3H_0^2 \approx 10^{-84}$ GeV^2 at the present epoch. This implies the reduction of zero modes through the singlet gauge boson condensation leading to the current anomaly. The effective vacuum energy density is connected with the effective cosmological constant by $V_e(\bar{\phi}) = \langle\rho_m\rangle_e = M_G^4 = \Lambda_e/8\pi G_N$, which shows the important role of the gauge boson mass M_G: as a function of the singlet graviton condensation $\bar{\phi} = \langle\phi\rangle$ parameterized by a scalar field, the effective potential is subtracted by $V(\bar{\phi}) = \frac{\Lambda}{8\pi G_N} = -2M_{Pl}^2 g_f g_g^2 \langle\phi\rangle^2 + g_f^2 g_g^4 \langle\phi\rangle^4 = \mu^2\langle\phi\rangle^2 + \lambda\langle\phi\rangle^4$ according to equations (9) and (12). The condensation of the singlet gauge boson, represented by the bare cosmological constant, cancels the vacuum energy and makes the accelerating expansion of the universe. At the Planck energy, the effective vacuum energy density $\langle\rho_m\rangle_e$ has the energy density of 10^{76} GeV^4 but below the Planck energy, the condensation of singlet gauge fields decreases it to the present value 10^{-47} GeV^4: $\langle\rho_m\rangle_e \approx 10^8$ GeV^4 at the weak energy and $\langle\rho_m\rangle_e \approx 10^{-4}$ GeV^4 at the QCD energy. Since $\bar{\phi} \approx 0$ above the Planck scale but $\bar{\phi} \approx 10^{19}$ GeV at the present universe scale, the discrepancy in the vacuum energy density leading to 10^{122} GeV^4 is consistent with the theoretical large value of the effective vacuum energy density at the Planck scale and the experimental small value at the present universe scale. It is realized that the total conserved particle number of gauge bosons is 10^{91}, the energy per gauge boson is 10^{19} GeV at the Planck epoch, and the energy per gauge boson is 10^{-12} GeV at the present epoch; the static maximum (minimum) gauge boson energy is 10^{19} GeV (10^{-12} GeV) at the Planck epoch and is 10^{-12} GeV (10^{-42} GeV) at the present epoch.

The gauge boson number density is given by $n_G = M_G^3$: $n_{Pl} \approx 10^{57}$ GeV$^3 \approx 10^{98}$ cm^{-3} at the Planck scale, $n_{EW} \approx 10^6$ GeV$^3 \approx 10^{47}$ cm^{-3} at the weak scale, $n_{QCD} \approx 10^{-2}$ GeV$^3 \approx 10^{39}$ cm^{-3} at the strong scale, and $n_0 \approx 10^{-36}$ GeV$^3 \approx 10^5$ cm^{-3} at the present scale. Under the constraint of the extremely flat universe, the relation $\Omega - 1 = (\langle\rho_m\rangle - \Theta\rho_m)/\rho_G - 1 = -10^{-61}$ leads to the Θ constant $\Theta = 10^{-61}\ \rho_G/\rho_m$, which will be further discussed. If the matter density in the universe is $\rho_m \simeq \rho_c \simeq 10^{-47}$ GeV^4 and is conserved, the Θ constant depends on the gauge boson mass M_G: $\Theta = 10^{-61}\ M_G^4/\rho_c$. Θ values becomes $\Theta_{Pl} \approx 10^{61}$, $\Theta_{EW} \approx 10^{-4}$, $\Theta_{QCD} \approx 10^{-10}$, and $\Theta_0 \approx 10^{-61}$ at different stages. This is consistent with the observed results, $\Theta < 10^{-9}$ in the electric dipole moment of the neutron [21] and $\Theta \simeq 10^{-3}$ in the neutral kaon decay [22] as CP violation parameters.

The effective cosmological constant corresponds to the gauge boson mass, which is large in the early small system before phase transition but is small in the later large system after phase transition: the nearly zero cosmological constant represents the nearly massless gauge boson responsible for the universe expansion discussed in the following section. The DSSB of gauge symmetry and discrete symmetries might give rise to the current anomaly like the axial current anomaly in QCD [11]. The true vacuum as the physical vacuum is achieved from the normal vacuum, which possesses larger symmetry group than the physical vacuum, through DSSB. The instanton mechanism as the vacuum tunneling is expected in the Euclidean spacetime. In the universe evolution, the breaking of discrete symmetries, P, C, CP, and T, is expected due to the condensation of the singlet gauge boson and is closely relevant for the baryon asymmetry. The Θ vacuum term in (2) explicitly violates CP symmetry: $\Theta = 10^{-61}\rho_G/\rho_m$ in equation (5). According to the observation for the electric dipole moment of the neutron $d_n = 2.7 \approx 5.2 \times 10^{-16}\Theta\ e$ cm, $\Theta \leq 10^{-9}$ at the strong scale [21]. CP violation due to the Θ vacuum term makes the baryon asymmetry $\delta_B \approx 10^{-10}$ observed at present [23]. According to the baryon asymmetry, the fermion matter current J_μ is conserved at the Planck epoch but the antifermion current $\bar{J}_\mu$ is not conserved, that is, $\partial_\mu J_\mu = 0$ but $\partial_\mu \bar{J}_\mu = \frac{g_g^2}{16\pi^2} TrG^{\mu\nu}\tilde{G}_{\mu\nu}$: this makes the fermion asymmetry $\delta_f = \frac{N_f}{N_{t\gamma}} = \frac{10^{91}}{10^{88}} = 10^3$ with the fermion mass 10^{-12} GeV as discussed in the following section. The bare vacuum energy density $\langle\rho_m\rangle = 10^{76}$ GeV^4 at the Planck scale is relevant for the Θ vacuum, which represents the surface term because it is total derivative and the effective cosmological constant decreases as the system expands. The vacuum energy density does not affect the perturbative aspect but does affect the nonperturbative aspect of the system. The massive gauge boson with the Planck mass is transformed to the nearly massless gauge boson with the mass 10^{-12} GeV at present. Massless gauge bosons (photons) respectively appear during phase transition at the Planck epoch and at the present epoch: $E_\gamma \approx 10^{19}$ GeV and $E_\gamma \approx 3 \times 10^{-13}$ GeV respectively. DSSB due to the initially large cosmological constant Λ_e derives the inflation of the system in the order of 10^{30}, which make the system expand exponentially [20]. This is totally compatible with the observed expectation of the small effective cosmological constant 10^{-84} GeV^2 or energy density 10^{-47} GeV^4 at the very low energy scale of the present universe and with the theoretical expectation of the large effective cosmological constant or energy density 10^{76} GeV^4 at the Planck scale of the early universe. This scheme may accordingly resolve the cosmological constant problem and may be compatible with observations BUMERANG-98 and MAXIMA-1 [2].

C. Modification of General Relativity: Θ Constant and Λ Constant

The value of Θ also makes the relation of the matter energy density ρ_m to the effective cosmological constant

Λ_e because of the relation $\Lambda_e = 8\pi G_N M_G^4$:

$$\rho_m = \frac{\rho_G}{10^{61}\Theta} = \frac{M_G^4}{10^{61}\Theta} = \frac{\Lambda_e}{10^{61}\Theta 8\pi G_N} \quad (15)$$

where $\rho_m = T_{00}$ with the energy-momentum tensor $T_{\mu\nu}$. One possible extension of Einstein's field equation (11) is obtained by using the above relation:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G_N \tilde{T}_{\mu\nu} = -8\pi G_N (T_{\mu\nu} - g_{\mu\nu}10^{61}\Theta\rho_m) \quad (16)$$

or, under the assumption of relativistic matter particle $T/2 = \rho_m$,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G_N \tilde{T}_{\mu\nu} = -8\pi G_N (T_{\mu\nu} - \frac{10^{61}\Theta}{2}g_{\mu\nu}T) \quad (17)$$

where the first term of the right hand side stands for matter energy and the second term stands for vacuum energy. Since the vacuum energy density is $V(\bar{\phi}) = \rho_G = 10^{61}\Theta\rho_m$, (16) is the equivalent form with (13). The Θ constant thus plays the role relating two different worlds, the matter world and the vacuum world. Using Θ values, effective cosmological constants are obtained: $\Lambda_e = 8\pi G_N M_G^4 = 10^{38}$ GeV^2 at the Planck epoch, $\Lambda_e = \Lambda_{EW} \approx 10^{-30}$ GeV^2 at the weak epoch, $\Lambda_e = \Lambda_{QCD} \approx 10^{-42}$ GeV^2 at the strong epoch, and $\Lambda_e = \Lambda_0 = 3H_0^2 \approx 10^{-84}$ GeV^2 at the present epoch. At the present universe with $\Theta_0 = 10^{-61}$, the modified general relativity (17) becomes

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G_N (T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) \quad (18)$$

which is identical with general relativity (13) since $T/2 = \rho_m = \Lambda_0/8\pi G_N \simeq \rho_c$. Note that the left hand side and the right hand side have completely symmetric forms in two tensors $R_{\mu\nu}$ and $T_{\mu\nu}$.

V. CLASSICAL TESTS

In the previous sections, Newton gravitation constant is defined as the effective coupling constant and the effective cosmological constant is connected to the effective vacuum energy due to massive gauge bosons. In this section, only classical tests for QG are concentrated while quantum tests for QG are separately discussed in subsequent papaer [3].

Whatever the gauge group G for QG is, a gauge theory for gravity can exhibit classical tests verified by Einstein's general relativity in the weak field limit [9]. Based on QG, the behavior of the gauge boson depends on its energy compared with its mass. If the gauge boson has energy $E > M_{Pl}$, the gravitational propagation becomes like $e^{ik(r-l_{Pl})}$. If $E < M_{Pl}$, propagation $e^{-M_G^e(r-l_{Pl})}$ is limited within the Planck scale $l_{Pl} \approx 1/M_{Pl}$ scale; this is the Yukawa interaction with the massive graviton. When $E \to M_{Pl}$, the massive gauge boson mediates the constant gravitational potential. When $E < M_{Pl}$, the massive gauge boson mediates the Yukawa type potential $e^{-M_G^e(r-l_{Pl})}$ where quadrupole moment interactions lead to Einstein's general relativity. Then the Newtonian approximation limit of general relativity is the familiar Newtonian mechanics in the macroscopic scale. The gauge boson with the energy less than the Planck mass provides Newtonian mechanics outside the Planck scale; the relativistic correction to the rest mass is the direct reason for the modification of Newtonian mechanics as the quadrupole moment interaction. Newtonian mechanics in the solar system is the case of the large baryon number ($B = N_B >> 1$): in the relation between the gauge boson and fermion mass $M_G = mg_f g_g^2\sqrt{N_{sd}}$, $N_{sd} << 1$ in the case of $m >> M_G$.

A. Relations among Quantum Gauge Theory, General Relativity, and Newtonian Mechanics

QG based on quantum mechanics, gauge invariance, and special relativity is connected with general relativity based on equivalence principle and general coordinate invariance through the parameter Θ and with Newtonian mechanics in terms of the multipole expansion of the Yukawa gravitational potential. The brief outline is given as follows.

1. Relation between Quantum Gauge Theory and General Relativity

According to QG, the Θ vacuum generates matter as the result of the violation of discrete symmetries during DSSB: the parameter Θ provides the relation between the vacuum and matter as indicated by the baryon asymmetry. The explicit relation between QG and Einstein's generality is given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G_N \tilde{T}_{\mu\nu} = -8\pi G_N (T_{\mu\nu} - \frac{10^{61}\Theta}{2}g_{\mu\nu}T). \quad (19)$$

The weak field limit of general relativity can be, apart from the vacuum term $\Lambda = \Theta 10^{61} 8\pi G_N T/2$, obtained by decomposing $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $h_{\mu\nu} << 1$:

$$\Box h_{km} = -16\pi G_N (T_{km} - \eta_{km}T/2),$$
$$\Box (h^i_j - \frac{1}{2}\delta^i_j h) = -16\pi G_N T^i_j \quad (20)$$

where the first wave equation describes gravitational radiation as well as the response of the gravitational field to the source $T_{\mu\nu}$. In the weak field limit of Einstein's generality relativity, the matter interaction is postulated as the quadrupole (tensor) interaction with the angular momentum $l = 2$ (or $s = 2$), which possesses complete symmetric configuration. The energy is the source in general relativity, which mediates only attractive force. This information is consistent with the interpretation of QG since the gravitational electric quadrupole moment must be of even parity for matter and be of attraction: the odd parity for gravitational magnetic quadrupole moment disappears so as to conserve parity operation for matter. Massive gauge bosons with an $s = 2$ multiplet are $A_4 \sim A_8$ if QG is an $SU(3)$ gauge theory. A massive gauge boson with $s = 2$ has the mass $M_G \approx 10^{19}$ GeV at the Planck scale and the mass $E_\gamma \approx 10^{-12}$ GeV at the present scale. The massless gauge boson with $l = 2$ (or $s = 2$) in general relativity, conventionally regarded as the graviton, is realized as the massless photon with $l = 2$ (or $s = 2$) in QG in this aspect. The massless gauge boson with $s = 1$ or $s = 2$, known as the photon, has the energy $E_\gamma \approx 10^{18}$ GeV at the Planck scale and the energy $E_\gamma \approx 10^{-13}$ GeV as CMBR at the present scale.

2. Relation between General Relativity and Newtonian Mechanics

Newtonian mechanics is known as the weak field approximation or the Newtonian approximation of general relativity. Newtonian approximation is specified by the following assumptions. The motion of particles are nonrelativistic ($v << c$), the gravitational fields are weak in the sense that $g_{ij} = \eta_{ij} + h_{ij}$ with $h_{ij} << 1$, and the fields change slowly with the time. The inequality suggests that powers of h_{ij} higher than 2 in the action principle and higher than 1 in the field equations are neglected and time derivatives are ignored in comparison with space derivatives. In the quadrupole (tensor) interaction in the weak field limit, the mass of particle is the source, the gravitational potential $\phi = h_{00}/2$ is the Coulomb type potential ($\sim 1/r$), and there is only attractive force: $\nabla^2\phi = -4\pi G_N \rho_m$ or $\phi = -G_N m/r$ is derived from (20).

3. Relation between Quantum Gauge Theory and Newtonian Mechanics

The spherical mass is roughly regarded as the gravitational intrinsic electric monopole. The generation of the matter mass m is closely related to the spontaneous breaking of gauge and chiral symmetries. As the temperature of the system decreases, the gauge boson mass decreases due to the condensation of the singlet gauge boson and the matter particle mass m increases with the decrease of the difference number N_{sd}. It is realized that the Yukawa type gravitational potential $\phi(r) = e^{-M_G^e(r-l_{Pl})}$ is applied when $M_G^{e2} = (M_{Pl}^2 - E^2) > 0$ but the plain wave propagation $\phi(r) = e^{ik(r-l_{Pl})}$ is applied when $k^2 = (E^2 - M_{Pl}^2) > 0$.

Newtonian mechanics is the Yukawa gravitational potential associated with massive gauge boson: $M_G^e \simeq M_G \simeq M_{Pl}$ at the present universe. Using its multipole expansion around the boundary $l_{Pl} = 1/M_{Pl}$, the gravitational potential

$$\phi(r) = e^{-M_G(r-l_{Pl})} \tag{21}$$

becomes

$$\phi(r) = -\frac{1}{M_G r} + \frac{2}{M_G^2 r^2} \cdots \tag{22}$$

as the regular solution for multipole terms outside the Planck scale $r > l_{Pl}$ and becomes

$$\phi(r) = 1 - M_G r + M_G^2 r^2/2 \cdots \tag{23}$$

as the regular solution for multipole terms inside the Planck scale $r < l_{Pl}$. The quadrupole expansion ($l = 2$ or $s = 2$) in (22) is related to the Newtonian potential outside the Planck scale l_{Pl}. Newton gravitational constant G_N is the effective coupling constant specified by the mass of the gauge boson M_G: $G_N = \sqrt{2} g_f g_g^2/8M_G^2$. It is assumed that the interaction distance between constituent particles in analogy with the Bohr radius is expressed by $r_i = 1/m_i g_f \alpha_g$ with the constituent particle mass m_i and the macroscopic distance is expressed by $r = 1/m g_f \alpha_g \simeq 2\sqrt{2}/\pi \sum_i r_i$ with the total mass given by the sum of constituent masses $m = \sum_i m_i$. Then the collective quadrupole term is recovered as the Newtonian gravitational potential

$$\phi(r) = -G_N m/r \tag{24}$$

and the gravitational potential energy

$$U(r) = -G_N m_a m_b/r \tag{25}$$

where m_a and m_b are the masses of particles. Note that the quadrupole term of the potential (10) leads to the gravitational potential energy (25) outside the Planck scale. The matter mass m is connected with the gauge boson mass by $M_G = \sqrt{\pi} m g_f \alpha_g \sqrt{N_{sd}}$ with the difference number N_{sd} in intrinsic two-space dimensions. It is realized that since the force in Newtonian mechanics is $F = -G_N m_a m_b/r^2 \sim -1/N_{sd} r^2$, the great number of gravitons with the mass M_{Pl} are, roughly $1/N_{sd} \approx 10^{70}$ in the solar system, involved in gravitational interactions because, for example, the sun has the mass $m_S \sim 10^{57}$ GeV and the earth has the mass $m_E \sim 10^{51}$ GeV. Newtonian mechanics is the residual interaction of the graviton exchange just as the nuclear interaction is the residual interaction of the gluon exchange. This suggests that gravitational dipoles known as the Cooper pairs are formed and they do the quadrupole (tensor) interaction, which is approximated by the Coulomb type potential outside the Planck scale in the special case of the $z \approx 0$ plane.

B. Classical Test of Quantum Gravity

QG may be related to general relativity in the weak field limit and Newton mechanics in terms of gauge bosons with the energy less than the Planck mass. The energy per mass derived from QG is connected with Newtonian mechanics and its relativistic correction so that QG may also derive classical tests verified by general relativity in the weak field limit.

1. Newtonian Mechanics and Relativistic Correction

The collective quadrupole (tensor) interaction with $l = 2$ (or $s = 2$) is the matter interaction: this is the gravitational electric quadrupole to satisfy the parity operation of the matter space. Gravitational magnetic dipole is possible but gravitational electric dipole is not possible due to parity condition. Gravitational magnetic monopole violates parity operation and this might be the reason for no detection of magnetic monopole in the universe as indicated by the dual Meissner effect.

Based on the dual Meissner effect in gravitation, the graviton with the energy lower than the Planck mass can not propagate outside the Planck scale and the gravitational electric field is excluded in the gravitational magnetic superconducting state. When matter particles interact by the exchange of gravitons, their quadrupole interactions hold even parity outside the Planck scale. The small correction to Newtonian mechanics comes from the special relativistic effect to the rest matter mass m, which becomes the effective mass $m/\sqrt{1-u^2}$ with the velocity $\vec{u}$: $m \to m/(1-u^2/2) \simeq m(1+u^2/2)$ if $u << 1$. The gravitational potential energy in (25) leads to

$$U(r) = -G_N \frac{m_a m_b}{r}(1+u^2) \tag{26}$$

Note that the longitudinal component of $\vec{u}$ gives repulsion and the transverse component of $\vec{u}$ gives attraction.

To apply the above description to the solar system, assume that an object with the mass $m_a = m$ orbits around a very massive object with $m_b = M$. The energy per mass including the kinetic energy and potential energy can be written by

$$\begin{aligned} \varepsilon &= \frac{1}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) - r^2\dot{\varphi}^2\frac{G_N M}{r} - \frac{G_N M}{r} \\ &= \frac{1}{2}(\dot{r}^2 + r^2\dot{\varphi}^2[1 - 2\frac{G_N M}{r}]) - \frac{G_N M}{r} \end{aligned} \tag{27}$$

where the first two terms are the kinetic energies of the mass m in the polar coordinates and the last two terms are the Newtonian potential and its relativistic correction. Note that $\vec{u}$ have only the transverse components and the transverse particle velocity $u_\perp = r\dot{\varphi}$ is used in the above. In general relativity, total energy per mass is exactly identical to equation (27).

This verifies that gravity may be described by a gauge theory since both general relativity in the weak field limit and gauge theory with the DSSB mechanism can produce the identical energy per mass. The effective spin 2 interaction in general relativity is verified as the intrinsic or extrinsic quadrupole interaction ($s = 2$ or $l = 2$) with the effective coupling constant G_N in gauge theory. Equation (27) may provide explanations to the typical classical tests, the precession of the perihelion and the deflection of light [24].

VI. COMPARISON BETWEEN QUANTUM GRAVITY AND GENERAL RELATIVITY

This section is devoted to summarize and to convince QG as an $SU(N)$ gauge theory beyond general relativity: classical tests for QG are discussed in the previous section and quantum tests for QG are described in subsequent papaer [3].

The potential QG as a gauge theory may resolve serious problems of GUTs and the standard model: different gauge groups, Higgs particles, the inclusion of gravity, the proton lifetime, the baryon asymmetry, the family symmetry of elementary particles, inflation, fermion mass generation, etc. This scheme may also provide possible resolutions to the problems of Einstein's general relativity or the standard hot big bang theory: the spacetime singularity, cosmological constant, quantization, baryon asymmetry, structure formation, dark matter, flatness of the universe, and renormalizability, etc. Furthermore, the typical predictions of QG are compatible with recent experiments, BUMERANG-98 and MAXIMA-1 [2]: the flat universe, inflation, vacuum energy, dark matter, repulsive force, CMBR, etc.

Comparison between QG and general relativity is summarized by Table II. QG is quantum theory holding principles of gauge invariance and special relativity while Einstein's general theory of relativity is classical theory holding principles of equivalence and general relativity. Newly introduced concepts in QG are the flat universe rather than the curved universe, dynamical spontaneous symmetry breaking (DSSB) rather than spontaneous symmetry breaking, gauge group hierarchy $G \supset H \supset SU(2)_L \times U(1)_Y \times SU(3)_C$, coupling constant hierarchy $(\alpha, \alpha/3, \alpha/4, \alpha/12, \alpha/16)$ for both weak and strong interactions, effective coupling constant hierarchy $G_N \supset G_F \supset G_R$, effective cosmological constant $\Lambda_e = 8\pi G_N M_G^4$, massive gauge boson mass square $M_G^2 = M_{Pl}^2 - g_f g_g^2 \langle\phi\rangle^2 = g_f g_g^2 [A_0^2 - \langle\phi\rangle^2]$, massless gauge bosons as Nambu-Goldstone bosons including cosmic microwave background radiation [25], the dynamical spontaneous breaking of discrete symmetries (C, P, T, and CP), Newtonian mechanics as quadrupole (tensor) interactions, the possible modification of Einstein's general relativity, etc. Quantum tests will be discussed in subsequent paper [3] are strongly interacting massive particles in addition to strongly interacting massive particles as the candidate of dark matter, the universe inflation with the order of magnitude 10^{30}, the baryon asymmetry $\delta_B \approx 10^{-10}$, the relation between time and gauge boson mass, the internal and external quantization of space-time, proton decay time much longer than one predicted by GUTs, mass generation mechanism with the surface effect, etc. The gauge boson mass $M_G \simeq 10^{-12}$ GeV associated with the cosmological constant $\Lambda_0 \simeq 10^{-84}$ GeV^2 and the Hubble constant $H_0 \simeq 10^{-42}$ GeV especially indicates gauge theories for new fundamental forces responsible for the universe expansion and CMBR. The repulsive force at the present universe is recently suggested in BUMERANG-98 and MAXIMA-1 experiments [2].

VII. CONCLUSIONS

This study toward quantum gravity (QG) proposes an $SU(N)$ gauge theory with the Θ vacuum term as a trial theory, which suggests that a certain group G for gravitational interactions leads to a group $SU(2)_L \times U(1)_Y \times SU(3)_C$ for weak and strong interactions through dynamical spontaneous symmetry breaking (DSSB) leading to a current anomaly; the group chain is $G \supset SU(2)_L \times U(1)_Y \times SU(3)_C$. DSSB consists of two simultaneous mechanisms; the first mechanism is the explicit symmetry breaking of gauge symmetry, which is represented by the gravitational factor g_f and the gravitational coupling constant g_g, and the second mechanism is the spontaneous symmetry breaking of gauge fields, which is represented by the condensation of singlet gauge fields. Newton gravitation constant G_N originates from the effective coupling constant for massive gravitons, $\frac{G_N}{\sqrt{2}} = \frac{g_f g_g^2}{8M_G^2}$ with $M_G = M_{Pl} \approx 10^{19}$ GeV: the effective coupling constant chain is $G_N \supset G_F \times G_R$ for gravitation, weak, and strong interactions respectively. This scheme relates the effective cosmological constant to the effective vacuum energy associated with massive gauge bosons, $M_G^2 = M_{Pl}^2 - g_f g_g^2 \langle\phi\rangle^2$, and provides a plausible explanation for both the present small and the early large value of the cosmological constant; the condensation of the singlet gauge field $\langle\phi\rangle$ induces the current anomaly and subtracts the gauge boson mass as the system expands. This proposal thus suggests a viable solution toward such longstanding problems as the quantization of gravity and the cosmological constant. The crucial point is that DSSB mechanism is adopted to all the interactions characterized by gauge invariance, physical vacuum problem, and discrete symmetry breaking. In this approach, DSSB is currently underway in the evolution of the flat universe and Newton gravitation coupling constant G_N represents the massive gauge boson propagation just as Fermi electroweak coupling constant G_F does. Newton gravitation constant has the dimension of inverse energy square due to massive gravitons. A gauge theory allows to have the masses of gauge bosons without spoiling the renormalizability; the renormalizability of a gauge theory with DSSB was demonstrated by 't Hooft. The dimensionality of the effective gravitation coupling constant, which prevents gravity from the renormalizability, may be removed in terms of the concept of DSSB. The compatibility between gravitation theory and gauge theory may be extended to the quantization and renormalization of gravity and the non-renormalizability problem of gravity may be resolved in the potential gauge theory for gravity.

This study claims that the effective cosmological constant defined by $\Lambda_e = 8\pi G_N M_G^4$ is related to the effective vacuum energy $V_e(\bar{\phi}) = \langle\rho_m\rangle_e = M_G^4$ represented by the gauge boson mass M_G, which plays an important role specially at the very early and later stage of gravitational evolution: $\Lambda_e \approx 10^{38}$ GeV^2 or $\langle\rho_m\rangle_e \approx 10^{76}$ GeV^4 at the Planck epoch and $\Lambda_e = \Lambda_0 \approx 10^{-84}$ GeV^2 or $\langle\rho_m\rangle_e = \langle\rho_m\rangle_0 \approx 10^{-47}$ GeV^4 at the present epoch. This proposal may resolve the cosmological constant problem by interpreting that the condensation of singlet gauge bosons exactly cancels the bare vacuum energy density $\langle\rho_m\rangle$ due to the condensation of fermions through the universe expansion. The effective vacuum energy linked by the gauge boson mass becomes significant during phase transition in the early universe and makes the universe expand exponentially. This scheme is therefore consistent with both the experimental expectation of the very small effective cosmological constant at the present epoch and the theoretical expectation of the very large cosmological constant at the early universe; the condensation of singlet gauge bosons makes gauge bosons lighter as the universe expands and phase transition takes place. The Θ constant is parameterized by $\Theta = 10^{-61}\ \rho_G/\rho_m = 10^{-61}\ \Lambda_e/8\pi G_N \rho_m$ with the vacuum energy density $\rho_G = M_G^4$ and the matter energy density $\rho_m \simeq \rho_c \simeq 10^{-47}$ GeV^4. This suggests the possible modification of general relativity in terms of the Θ constant rather than the cosmological constant Λ: $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G_N(T_{\mu\nu} - \frac{10^{61}\Theta}{2}g_{\mu\nu}T)$.

A QG is consistent with general relativity in the weak field limit as well as Newtonian mechanics since classical tests can be successfully shown from the gauge theory point of view. The relation between QG and Einstein's general relativity is explicitly given in terms of cosmological parameters like Λ and Θ. The graviton with the energy lower than the Planck mass has the Yukawa type potential $\phi(r) = e^{-M_G^e(r-l_{Pl})}$ with the effective gauge boson mass $M_G^e = ik_G = (M_G^2 - E^2)^{1/2}$ while the graviton with the energy equal to the Planck mass has the Coulomb potential energy $-G_N m^2/r$. This implies the effective interaction of the graviton below the Planck energy is proportional to $G_N J_\mu J^{\mu\dagger}$ in the low momentum transfer. When $E < M_{Pl}$, the massive gauge boson mediates the Yukawa type potential $\phi(r) = e^{-M_G^e(r-l_{Pl})}$ where the quadrupole moment term leads to the weak field approximation of Einstein's general relativity: $\phi(r) = h_{00}/2$ with $g_{ij} = \eta_{ij} + h_{ij}$ and $h_{ij} << 1$. Newtonian mechanics, which is the Newtonian approximation of the weak field limit of general relativity, is interpreted as collective quadrupole (tensor) interactions of gravitons with the energy less than the Planck mass ($E < M_{Pl}$). The energy per mass obtained from this scheme, which has the Newtonian potential for the rest mass and the relativistic correction to the rest mass, is the same with one obtained from general relativity; the energy per mass can leads to the classical tests verified by general relativity. This work might accordingly give rise to a turning point toward the unification of fundamental forces in nature. QG as a potential gauge theory may resolve the longstanding problems, the quantization of gravity and the cosmological constant problem. This proposal beyond the standard model thus implies that a potential, ultimate theory of nature may be found in the form of quantum field theory.

Notable accomplishments of this work are summarized as follows. A potential QG as a gauge theory is introduced and its relation to Einstein's general relativity is suggested. The origin of Newton gravitation constant G_N is illustrated as the Yukawa coupling of the massive graviton. The origin of the cosmological constant is given as the vacuum energy density represented by the gauge boson mass; the total gauge boson number 10^{91} in the universe is predicted as a conserved good quantum number. The DSSB of local gauge symmetry and global chiral symmetry triggers the baryon current anomaly. Newton gravitation constant G_N as the effective coupling constant provides the construction of the dimensionless coupling constant through the condensation of singlet gauge fields, which makes this theory renormalizable. The classical tests can be shown as the illustration of QG as a gauge theory even though the exact group for QG is not uncovered yet. This scheme may also provide possible resolutions to the problems of Einstein's general relativity or the standard hot big bang theory: the spacetime singularity, cosmological constant, quantization, baryon asymmetry, structure formation, dark matter, flatness of the universe, and renormalizability, etc. This work significantly contributes to the unification of fundamental forces from the viewpoint of the gauge field theories since all the known forces might be formulated in terms of gauge theories; the standard hot big bang theory with many pros and cons [26,27] may be replaced by this scheme toward the theory of everything.

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TABLE I. Features of Fundamental Interactions

Feature	Gravity	Electromagnetic	Weak	Strong
Gauge Boson	Graviton	Photon	Intermediate Boson	Gluon
Source	Spin (?)	Electric Charge	Isospin	Color
Coupling Constant	$\alpha_g(M_{Pl}) \simeq 0.02$ (?)	$\alpha_e \simeq 1/137$	$\alpha_i(M_Z) \simeq 0.12$	$\alpha_s(\Lambda_{QCD}) \simeq 0.48$
Gauge Boson Mass	10^{19} GeV	0	10^2 GeV	10^{-1} GeV
Effective Coupling	$G_N \simeq 10^{-38}$ GeV^{-2}		$G_F \simeq 10^{-5}$ GeV^{-2}	$G_R \simeq 10^1$ GeV^{-2}
Cross Section	10^{-70} m^2	10^{-33} m^2	10^{-44} m^2	10^{-30} m^2
Lifetime	10^{57} s	10^{-20} s	10^{-8} s	10^{-23} s

TABLE II. Comparison between Quantum Gravity and General Relativity

Classification	QG	General Relativity
Exchange Particles	massive gravitons	massless gravitons
DSSB	yes	no
Discrete symmetries (P, C, T, CP)	breaking	no
Monopole Confinement	yes	unknown
Cosmological constant	$\Lambda_e = 8\pi G_N M_G^4$	$\Lambda_0 = 10^{-84}$ GeV^2
Inflation	10^{30}	no
Matter mass generation	$\rho_m = 10^{-61}\rho_G\Theta$	unknown
Baryogenesis	10^{78} for 0.94 GeV proton	no
Leptogenesis	10^{81} for 0.5 MeV electron	no
Proton decay time	10^{57} s	unknown
Singularity	no	yes
Universe	flat ($\Omega = 1 - 10^{-61}$)	curved
Dark matter	yes	unknown
Interactions	monopole or dipole	quadrupole (tensor)
Renormalization	yes	no
Relativity	special	general
Principle	gauge invariance	equivalence
Free parameter	coupling constant (α_g)	effective coupling constant (G_N)